# Supplement to

Metzger, S., Junkermann, W., Butterbach-Bahl, K., Schmid, H. P., and Foken, T.:
**Measuring the 3-D wind vector with a weight-shift microlight aircraft**

It is the intention of this supplement to make transparent the procedures in the main article, and to guide the reader through the relevant calculations. Supplement A provides the formulary necessary to compute the wind vector from a weight-shift microlight aircraft. A model to propagate uncertainty through the wind vector equations is provided in Supplement B1. Relevant notation and abbreviations are listed in Supplement C. References to literature are given at the end of the document.

## Supplement A Wind measurement transformation equations

The wind measurement from aircraft requires several coordinate systems, as well as angles to transform between them (Fig. 2). We define the wind vector $\boldsymbol{v}^{\mathrm{m}} = (v_{\mathrm{u}}^{\mathrm{m}}, v_{\mathrm{v}}^{\mathrm{m}}, v_{\mathrm{w}}^{\mathrm{m}})$ in the standard meteorological coordinate system (MCS, superscript m, positive eastward, northward, and upward). Then $\boldsymbol{v}^{\mathrm{m}}$ can be calculated from navigation, flow and attitude measurements: In the MCS $\boldsymbol{v}^{\mathrm{m}}$ is expressed as the vector difference between the aircraft's ground speed vector ($\boldsymbol{v}_{\mathrm{gs}}^{\mathrm{m}}$), directly measured by the inertial navigation system (INS), and the true airspeed vector ($\boldsymbol{v}_{\mathrm{tas^m}}$), essentially measured by the five hole probe (5HP, Williams and Marcotte, 2000):

$$\begin{aligned} \boldsymbol{v}^{\mathrm{m}} &= \boldsymbol{v}_{\mathrm{gs}}^{\mathrm{m}} - \boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}} \\ &= \boldsymbol{v}_{\mathrm{gs}}^{\mathrm{m}} - \mathbf{M}^{\mathrm{bm}} \times \left( \mathbf{M}^{\mathrm{ab}} (-v_{\mathrm{tas}}) + \boldsymbol{v}_{\mathrm{lev}}^{\mathrm{b}} \right). \end{aligned} \tag{A1}$$

Yet the quantity directly measured by the 5HP is the true airspeed scalar $v_{\mathrm{tas}}$. The second, decomposed form of the wind vector Eq. (A1) indicates that several calculation steps are necessary to arrive at the desired vector quantity $\boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}}$.

In the following we will walk through these successive steps, starting with the 5HP measurements. From the ports of the 5HP (Fig. 3) three differential pressures were measured:

$$p_{\mathrm{q,A}} = p_{\mathrm{t}} - p_{\mathrm{s}}, \tag{A2}$$

$$p_{\alpha} = p_3 - p_1, \text{ and} \tag{A3}$$

$$p_{\beta} = p_4 - p_2. \tag{A4}$$

Measured dynamic pressure $p_{\mathrm{q,A}}$ (subscript upper-case letters A–G indicate calibration stage), and attack- and sideslip differential pressures $p_{\alpha}$, $p_{\beta}$ were used to calculate the airflow angles (Williams and Marcotte, 2000):

$$\alpha_{\mathrm{A}} = \frac{2}{9\sin(2\tau)} \frac{p_{\alpha}}{p_{\mathrm{q,A}}}, \text{ and} \tag{A5}$$

$$\beta_{\mathrm{A}} = \frac{2}{9\sin(2\tau)} \frac{p_{\beta}}{p_{\mathrm{q,A}}}. \tag{A6}$$

Here $\tau = 45°$ is the angle between the central port $p_t$ and the other ports $p_1$ through $p_4$ on the 5HP half sphere. Defining the normalization factor $D = \sqrt{1 + \tan^2 \alpha_{\mathrm{A}} + \tan^2 \beta_{\mathrm{A}}}$ the measured dynamic pressure $p_{\mathrm{q,A}}$ can be corrected for the pressure drop occurring at elevated airflow angles:

$$p_{\mathrm{q,B}} = p_{\mathrm{q,A}} \left( \frac{9 - 5D^2}{4D^2} \right)^{-1}. \tag{A7}$$

Now we can derive $v_{\mathrm{tas}}$ from the thermodynamic measurements of the 5HP: Due to stagnation at the tip of the 5HP ambient air is heated from its intrinsic temperature ($T_{\mathrm{s}}$) to total temperature ($T_{\mathrm{t}}$). Assuming adiabatic heating, Bernoulli's equation

$$\begin{aligned} v_{\mathrm{tas}}^2 &= 2c_{\mathrm{p,h}}(T_{\mathrm{t}} - T_{\mathrm{s}}) \\ &= 2c_{\mathrm{p,h}} T_{\mathrm{s}} \left[ \left( \frac{p_{\mathrm{s}}}{p_{\mathrm{s}} + p_{\mathrm{q}}} \right)^{-\kappa} - 1 \right], \end{aligned} \tag{A8}$$

gives $v_{\mathrm{tas}}$ as a function of the temperature difference (Leise and Masters, 1993). Since $T_{\mathrm{t}}$ can not be measured directly, it is substituted in Eq. (A8) by the adiabatic process (ram rise)

$$T_{\mathrm{t}} = T_{\mathrm{s}} \left( \frac{p_{\mathrm{s}}}{p_{\mathrm{s}} + p_{\mathrm{q}}} \right)^{-\kappa}, \tag{A9}$$

with the Poisson number $\kappa = 1 - \frac{c_{\mathrm{v,h}}}{c_{\mathrm{p,h}}}$. Furthermore the wind measurement should be independent of air humidity (subscript h). Therefore the specific heats under constant pressure (subscript p) $c_{\mathrm{p,h}}$ or constant volume (subscript v) $c_{\mathrm{v,h}}$ of moist air have to be derived from the specific heat constants for dry air (subscript d) and water vapour (subscript w), $c_{\mathrm{p,d}} = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $c_{\mathrm{p,w}} = 1846\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $c_{\mathrm{v,d}} = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$, and $c_{\mathrm{v,w}} = 1384\,\mathrm{J\,kg^{-1}\,K^{-1}}$ (Khelif et al., 1999):

$$c_{\mathrm{p,h}} = c_{\mathrm{p,d}} \left[ 1 + q \left( \frac{c_{\mathrm{p,w}}}{c_{\mathrm{p,d}}} - 1 \right) \right],$$

$$c_{\mathrm{v,h}} = c_{\mathrm{v,d}} \left[ 1 + q \left( \frac{c_{\mathrm{v,w}}}{c_{\mathrm{v,d}}} - 1 \right) \right], \quad \text{with specific humidity being}$$

$$q = \varepsilon \frac{e}{p_{\mathrm{s}} + e(\varepsilon - 1)}, \tag{A10}$$

where $\varepsilon = 0.622$ is the ratio of molecular weight of water vapour to that of dry air, and $e$ is the 5HP measured water vapour pressure.

Once derived, the scalar quantity $v_{\mathrm{tas}}$ has to be transformed into a vector quantity. This can be achieved by defining the aerodynamic coordinate system (ACS, superscript a, positive forward, starboard, and downward), which has its origin at the 5HP tip. In this coordinate system the true airspeed vector has the components $\boldsymbol{v}^{\mathrm{a}}_{\mathrm{tas}} = (-v_{\mathrm{tas}}, 0, 0)$. Since the ACS is aligned with the streamlines its orientation however varies in time. Therefore $\boldsymbol{v}^{\mathrm{a}}_{\mathrm{tas}}$ is transformed into a fixed coordinate system, that is the trike body coordinate system (BCS, superscript b, positive forward, starboard, and downward) with its origin in the INS. This is accomplished by successive rotations about the vertical axis $\mathrm{Z^a}$ and the transverse axis $\mathrm{Y^a}$. Following Lenschow (1986) the rotations can be summarized in the operator

$$\mathbf{M}^{\mathrm{ab}} = D^{-1} \begin{pmatrix} 1 \\ \tan\beta \\ \tan\alpha \end{pmatrix}, \tag{A11}$$

with the 5HP derived airflow angles of attack $\alpha$ and sideslip $\beta$, and the normalization factor $D$ as derived in Eqs. (A5)–(A7). Since $\boldsymbol{v}_{\mathrm{tas}^{\mathrm{a}}}$ carries all its information in the first vector component, it is sufficient to apply this transformation to $-v_{\mathrm{tas}}$ in the wind vector Eq. (A1).

Now the wind vector is known in the orientation of the BCS, yet with its origin still at the 5HP tip as initially defined in the ACS. To allow for the vector difference as required in the wind Eq. (A1) we have to account for the displacement of ACS origin (5HP tip) relative to the BCS origin (INS). This is done by considering the lever arm correction vector (Williams and Marcotte, 2000):

$$\boldsymbol{v}^{\mathrm{b}}_{\mathrm{lev}} = \begin{pmatrix} \Omega^{\mathrm{b}}_{\Phi} \\ \Omega^{\mathrm{b}}_{\Theta} \\ \Omega^{\mathrm{b}}_{\Psi} \end{pmatrix} \times \begin{pmatrix} x^{\mathrm{b}} \\ y^{\mathrm{b}} \\ z^{\mathrm{b}} \end{pmatrix}, \tag{A12}$$

with INS measured body rates $\Omega^{\mathrm{b}}_{\Phi}$, $\Omega^{\mathrm{b}}_{\Theta}$, $\Omega^{\mathrm{b}}_{\Psi}$ about the $\mathrm{X}^{\mathrm{b}}$, $\mathrm{Y}^{\mathrm{b}}$, $\mathrm{Z}^{\mathrm{b}}$ axes, and the displacement of the 5HP with respect to the INS along these axes, $x^{\mathrm{b}} = -0.73\,\mathrm{m}$, $y^{\mathrm{b}} = -0.01\,\mathrm{m}$, and $z^{\mathrm{b}} = 0\,\mathrm{m}$. The vector sum $\mathrm{M}^{\mathrm{ab}}(-v_{\mathrm{tas}}) + \boldsymbol{v}^{\mathrm{b}}_{\mathrm{lev}}$ in the wind Eq. (A1) then describes the true airspeed vector in the BCS.

A last step remains to obtain $\boldsymbol{v}^{\mathrm{m}}_{\mathrm{tas}}$ for use in the wind Eq. (A1), that is the transformation of the true airspeed vector from the BCS into the MCS. This is achieved by a first transformation into the geodetic coordinate system (GCS, superscript g, positive northward, eastward and downward) via successive rotations about the $\mathrm{X}^{\mathrm{b}}$, $\mathrm{Y}^{\mathrm{b}}$, $\mathrm{Z}^{\mathrm{b}}$ axes (Lenschow, 1986). From there the wind vector is permutated into the MCS (positive eastward, northward and upward). The transformations can be summarized in the operator

$$\mathbf{M}^{\mathrm{bm}} = \begin{pmatrix} M_{11}^{\mathrm{bm}} & M_{12}^{\mathrm{bm}} & M_{13}^{\mathrm{bm}} \\ M_{21}^{\mathrm{bm}} & M_{22}^{\mathrm{bm}} & M_{23}^{\mathrm{bm}} \\ M_{31}^{\mathrm{bm}} & M_{32}^{\mathrm{bm}} & M_{33}^{\mathrm{bm}} \end{pmatrix}, \text{ with} \tag{A13}$$

$$\begin{aligned}
M_{11}^{\mathrm{bm}} &= \sin\Psi^{\mathrm{b}}\cos\Theta^{\mathrm{b}}, \\
M_{12}^{\mathrm{bm}} &= \cos\Psi^{\mathrm{b}}\cos\Phi^{\mathrm{b}} + \sin\Psi^{\mathrm{b}}\sin\Phi^{\mathrm{b}}\sin\Theta^{\mathrm{b}}, \\
M_{13}^{\mathrm{bm}} &= \sin\Psi^{\mathrm{b}}\sin\Theta^{\mathrm{b}}\cos\Phi^{\mathrm{b}} - \cos\Psi^{\mathrm{b}}\sin\Phi^{\mathrm{b}}, \\
M_{21}^{\mathrm{bm}} &= \cos\Psi^{\mathrm{b}}\cos\Theta^{\mathrm{b}}, \\
M_{22}^{\mathrm{bm}} &= \cos\Psi^{\mathrm{b}}\sin\Theta^{\mathrm{b}}\sin\Phi^{\mathrm{b}} - \sin\Psi^{\mathrm{b}}\cos\Phi^{\mathrm{b}}, \\
M_{23}^{\mathrm{bm}} &= \sin\Psi^{\mathrm{b}}\sin\Phi^{\mathrm{b}} + \cos\Psi^{\mathrm{b}}\sin\Theta^{\mathrm{b}}\cos\Phi^{\mathrm{b}}, \\
M_{31}^{\mathrm{bm}} &= \sin\Theta^{\mathrm{b}}, \\
M_{32}^{\mathrm{bm}} &= -\cos\Theta^{\mathrm{b}}\sin\Phi^{\mathrm{b}}, \\
M_{33}^{\mathrm{bm}} &= -\cos\Theta^{\mathrm{b}}\cos\Phi^{\mathrm{b}},
\end{aligned}$$

where $\Phi^{\mathrm{b}}$, $\Theta^{\mathrm{b}}$, and $\Psi^{\mathrm{b}}$ are the INS measured attitude angles roll, pitch and heading, respectively. Finally the movement of the BCS with respect to the MCS is described by $\boldsymbol{v}_{\mathrm{gs}}^{\mathrm{m}}$ in the wind vector Eq. (A1).

## Supplement B Uncertainty quantification

### B1 Uncertainty propagation

In Eq. (A1) the wind vector is the difference between the aircraft's ground speed vector ($\boldsymbol{v}_{\mathrm{gs}}^{\mathrm{m}}$) and the true airspeed vector ($\boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}}$). The measurement of $\boldsymbol{v}_{\mathrm{gs}}^{\mathrm{m}}$ is readily provided by the inertial navigation system, together with the related uncertainty (Table 2). Uncertainty propagation is however required for $\boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}}$, since 12 measured quantities are merged during its calculation. The magnitude of the lever arm correction Eq. (A12), and with it possible uncertainty from this source, is two orders lower than $\boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}}$. It can therefore be neglected in the uncertainty propagation, which leaves nine measured quantities. By preprocessing Eqs. (A5)–(A10) these are further condensed to three measured quantities and three derived variables (see next paragraph for respective uncertainty propagation). Modified after Vörsmann (1985) the input uncertainty of the $\boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}}$ measurement can then be calculated from a linearised uncertainty propagation model in the vector components $v_{\mathrm{tas,c}}^{\mathrm{m}}$ (c = u, v, or w):

$$\Delta v_{\mathrm{tas,c}}^{\mathrm{m}} = \sum_{i=1}^{i=6} \left| \frac{\delta v_{\mathrm{tas,c}}^{\mathrm{m}}}{\delta f_i} \, \sigma(f_i) \right|, \tag{B1}$$

with $\frac{\delta v_{\mathrm{tas,c}}^{\mathrm{m}}}{\delta f_i}$ being the partial derivatives of Eqs. (A11) and (A13) inserted into the wind vector Eq. (A1). Thereby the input uncertainty of $\boldsymbol{v}_{\mathrm{tas}}^{\mathrm{m}}$ can be expressed as function of the (assumed independent) input variables ($f_i$), with $\sigma(f_i)$ being their respective uncertainty. Here $f_i$ are three quantities directly measured by the INS (i.e. pitch- ($\Theta^{\mathrm{b}}$), roll- ($\Phi^{\mathrm{b}}$) and heading- ($\Psi^{\mathrm{b}}$) angles) and three variables derived from five hole probe measurements (i.e. attack angle ($\alpha$), sideslip angle ($\beta$) and true airspeed scalar ($v_{\mathrm{tas}}$)). Such a procedure is conservative, since it assumes uncertainty interference, but not cancellation. It yields the maximum possible uncertainty triggered by the combined effects of $\sigma f_i$. The derivatives were further simplified by small-angle approximation. This simplification allows to express the input uncertainty with sign and sensitivity as a function of $\Psi^{\mathrm{b}}$, whereas the full form yields the maximum absolute input uncertainty for different flight states.

In analogy uncertainty propagation models were formulated for the three derived variables $\alpha$ in Eq. (A5), $\beta$ in Eq. (A6) and $v_{\mathrm{tas}}$ in Eq. (A8). These permit to express the actual uncertainties originating from the six remaining directly measured quantities, i.e. both flow angle differential pressures, dynamic- and static pressures, static temperature, and water vapour pressure.

With this setup the overall uncertainty at each stage of the wind calculation procedure can be evaluated through Gaussian uncertainty propagation (e.g., Taylor, 1997):

$$\sigma_{\mathrm{gau}} = \sqrt{\sum_{i=1}^{N} \sigma_i^2}, \tag{B2}$$

with $N$ being the number of (assumed linear and independent) uncertainty terms contributing to the stage investigated.

### B2 Uncertainty measures

For applications in the atmospheric boundary layer the comparison to a reference standard can yield an integral measure of confidence under varying conditions (e.g., Vogt and Thomas, 1995; Mauder et al., 2006). Therefore this study employs two basic bivariate criteria for the comparison of wind components. These are the root mean square deviation (RMSE) and bias (BIAS) between sample and reference (ISO, 1993):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(A_i - R_i)^2}, \tag{B3}$$

$$\mathrm{BIAS} = \frac{1}{N}\sum_{i=1}^{N}(A_i - R_i), \tag{B4}$$

with $N$ being the number of data couples $R_i$ and $A_i$, $R_i$ being the $i$th reference observation and $A_i$ the $i$th observation by aircraft sensors, sampled simultaneously. RMSE is also called comparability and is a measure of overall uncertainty. BIAS is the systematic difference between the mean of the measurements and the reference. These criteria were not normalized, since no consistent dependence on the wind magnitude or the aircraft's true airspeed was found.

## Supplement C Notation

Scalars and vector components are displayed in italics, vectors are displayed in bold italics, and matrices are displayed in bold roman typeface, respectively. Where applicable coordinate systems and respective axes are indicated by superscripts, whereas subscripts are used as specifiers.

### C1 Operators

| | |
|---|---|
| $[\mathbf{M}]$ | Transformation matrix |
| $[\delta]$ | Differential operator |
| $[\Delta]$ | Difference operator |

### C2 Parameters and variables

| | |
|---|---|
| $a$ | Acceleration |
| $A$ | Aircraft measurement |
| BIAS | Bias |
| $CL$ | Lift coefficient |
| $c_\mathrm{p}$ | Specific heat at constant pressure |
| $c_\mathrm{v}$ | Specific heat at constant volume |
| $D$ | Derived term containing airflow angles |
| $f$ | Place-holder for input variables |
| $g$ | Gravitational acceleration |
| $i$ | Continuous index |
| $L$ | Lift |
| $LF$ | Loading factor |
| $\frac{L}{S}$ | Wing loading |
| $m$ | Mass |
| $n$ | Normalized centre of pressure – 5HP separation distance |
| $N$ | Sample size |
| $p$ | Pressure |

| | |
|---|---|
| $q$ | Specific humidity |
| $R$ | Reference measurement |
| RMSE | Root mean square error |
| $S$ | Wing surface area |
| $T$ | Temperature |
| $v$ | Velocity scalar or vector component |
| $\boldsymbol{v}$ | Velocity vector |
| $x$, $y$, $z$ | Distances on respective coordinate axes |
| $\frac{z}{L}$ | Stability parameter |
| $\alpha$ | Angle of attack |
| $\beta$ | Angle of sideslip |
| $\varepsilon$ | Ratio of molecular masses |
| $\Theta$ | Pitch |
| $\kappa$ | Poisson number |
| $\Phi$ | Roll |
| $\xi$ | Wing upwash direction |
| $\pi$ | Perimeter constant |
| $\rho$ | Air density |
| $\sigma$ | Standard deviation, RMSE |
| $\tau$ | Angle between central and surrounding ports on half-sphere |
| $\Psi$ | Heading |
| $\Omega$ | Body rate |

## C3 Subscripts – superscripts

| | |
|---|---|
| 1–4 | Pressure ports |
| $\infty$ | Free airstream |
| $+, -$ | Into wind, with wind |
| $\tilde{}$ | Wind tunnel |

| | |
|---|---|
| a | Aerodynamic coordinate system, positive forward, starboard, and downward |
| A–G | Calibration steps |
| b | Body coordinate system, positive forward, starboard and downward |
| d | Dry air |
| g | Geodetic coordinate system, positive northward, eastward and downward |
| gau | Gaussian uncertainty propagation |
| gs | Ground speed |
| h | Humid air |
| lev | Lever arm |
| m | Meteorological coordinate system, positive eastward, northward and upward |
| off | Offset |
| q | Dynamic- |
| r | Inverse reference |
| s | Static- |
| slo | Slope |
| t | Total- |
| tas | True airspeed |
| u, v, w | Wind components in x, y, z directions |
| up | Upwash |
| w | Water vapour; Wing coordinate system, positive forward, starboard and downward |
| x, y, z | Standard Cartesian coordinate axes |
| $\alpha$ | Angle of attack |
| $\beta$ | Angle of sideslip |

## C4 Abbreviations

| | |
|---|---|
| 5HP | Five hole probe |
| ABL | Atmospheric boundary layer |
| ACS | Aerodynamic coordinate system,<br>positive forward, starboard, and downward |
| a.g.l. | Above ground level |
| a.s.l. | Above sea level |
| BCS | Body coordinate system, positive forward, starboard and downward |
| D-MIFU | Name of aircraft |
| DAQ | Data acquisition |
| E | East |
| EC | Eddy covariance |
| EIDAS | Embedded Institute for Meteorology and<br>Climate Research data acquisition system |
| FWA | Fixed-wing aircraft |
| GCS | Geodetic coordinate system,<br>positive northward, eastward and downward |
| INS | Inertial navigation system |
| IU | Input uncertainty |
| LI | Lindenberg |
| MCS | Meteorological coordinate system,<br>positive eastward, northward and upward |
| N | North |
| S | South |
| ST | Lake Starnberg |
| ULS | Universal laser sensor |
| VW1–VW3 | Vertical wind specific flight patterns |
| W | West |

| | |
|---|---|
| WCS | Wing coordinate system,<br>positive forward, starboard and downward |
| WSMA | Weight-shift microlight aircraft |
| XI | Xilinhot |

## References


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