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50 YEARS OF THE MONIN–OBUKHOV SIMILARITY THEORY

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Abstract. This historical survey shows that Obukhov's 1946 fundamental paper on a universal length scale for exchange processes in the surface layer was the basis for the derivation of the similarity theory by Monin and Obukhov in 1954. A brief overview of the experiments and findings used to formulate the universal functions in the presently used form is given. Finally, the current status of the theory is described, covering topics such as the accuracy of the universal functions and the turbulent Prandtl number.

Keywords: Eddy covariance, History of micrometeorology, Monin–Obukhov similarity theory, Obukhov length, Surface layer, Universal function.

1. Introduction

After A. M. Obukhov found a universal length scale for exchange processes in the surface layer in 1946, a logical consequence was the derivation of the similarity theory by A. S. Monin and Obukhov (1954). This theory was the starting point for modern micrometeorology, including the development of new measuring devices and the execution of several important experiments.

On the occasion of the 100th anniversary of Reynolds' decomposition theorem (Reynolds, 1894), J. A. Businger gave an overview of the history of boundary-layer meteorology (micrometeorology) during the Tenth Conference on Turbulence and Boundary Layers (Portland, Oregon, U.S.A. in 1992). In line with that presentation and the survey by Lumley and Yaglom (2001), this paper will present a short survey that places the Monin–Obukhov similarity theory (Monin and Obukhov, 1954) in historical context. In addition, the present status of the theory is discussed.

My personal and professional contact with A. M. Obukhov and A. S. Monin developed during common expeditions beginning over 30 years ago. In 1975 and 1976 I was a member of the KASPEX-75 and KASPEX-76 expeditions (Foken et al., 1978) with the Institute of Oceanology, Moscow (director: A. S. Monin) and in 1981 I participated in the International Turbulence Comparison Experiment (ITCE-81) in Tsimlyansk, Russia (Tsvang

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et al., 1985) with the Institute of Physics of the Atmosphere (director: A. M. Obukhov). From 1980 to 1990, I was scientific secretary of the KAPG project (Commission of the Academies of the Socialist countries for complex investigations on planetary geophysics) "Atmospheric Boundary Layers" initiated by A. M. Obukhov (Foken and Bernhardt, 1994).

2. Historical Overview

At the beginning of the Twentieth century, much progress was made in hydrodynamics connected with the fundamental work of Taylor (1915), Richardson (1920), and Prandtl (1925). Before the Second World War, experimental studies were mainly carried out in German-speaking countries. In Vienna, Schmidt (1925) formulated the 'Austausch coefficient'; in Munich, Geiger (1927) summarized microclimatological works in his famous book (still in print) 'The Climate near the Ground" (Geiger et al., 1995); in Leipzig, investigations of atmospheric turbulence were conducted (Lettau, 1939); and in Potsdam, after many experimental studies (Kleinschmidt, 1935), Albrecht wrote the first paper on the energy balance at the Earth's surface (Albrecht, 1940).

Following fundamental studies on turbulence spectra (Kármán and Howarth, 1938; Taylor, 1938), enquiries were continued by Russian scientists during the Second World War, primarily by Kolmogorov (1941a, b). Obukhov (1946, see Businger and Yaglom 1971; Obukhov, 1971) described work carried out in 1943 on the scaling length for the surface layer, later the basis of the Monin–Obukhov similarity theory (1954). In the early 1950s, many famous experiments, including these of Lettau, Swinbank, Dyer, and Tsvang, were conducted. In addition, the idea of a direct measuring method for turbulent fluxes, now known as the eddy covariance, was developed (Montgomery, 1948; Obukhov, 1951; Swinbank, 1951). This method became truly established after the development of the sonic anemometer, for which the basic equations are given by Schotland (1955). After the development of a sonic thermometer (Barrett and Suomi, 1949) during the O'Neill experiment in 1953 (Lettau, 1957), a vertical sonic anemometer with 1-m path length (Suomi, 1957) was used. The design of today's anemometers was developed by Bovscheverov and Voronov (1960), and later on by Kaimal and Businger (1963) and Mitsuta (1966). These phase shift anemometers have now been replaced by running time anemometers with time measurements (Hanafusa et al., 1982). Following early work by Sheppard (1947) the surface stress was directly measured with a drag plate in Australia (Bradley, 1968), and the sensible and latent heat fluxes were measured with highly sensitive, modified classical sensors (Dyer et al., 1967).

TABLE I

Important micrometeorological experiments up to 1986 (McBean et al., 1979; Foken, 1990; Garratt and Hicks, 1990), ITCE: International Turbulence Comparison Experiment.

Year	Place	Surface	Type, name	Reference
1953	O'Neill, U.S.A	Step	Boundary-layer exp.	Lettau (1957)
1962	Kerang, Australia	Step	Surface layer exp.	Swinbank and Dyer (1968)
1964	Hay, Australia	Step	Surface layer exp.	As above
1965	Hanford, U.S.A.	Sage	Anemometer comp.	Businger et al. (1969)
1967	Wangara experiment, Hay, Australia	Step	Surface and boundary layer exp.	Hess et al. (1981)
1968	Kansas, U.S.A.	Step	Micrometeorol. exp.KANSAS 1968	Izumi (1971)
1968	Vancouver, Canada	Water	ITCE-1968	Miyake et al. (1971)
1970	Tsimlyansk, Russia	Step	ITCE-1970	Tsvang et al. (1973)
1974	Koorin, Australia	Rough surface	Surface and boundary layer exp.	Garratt (1980)
1976	Conargo, Australia	Step	ITCE-1976	Dyer et al. (1982)
1981	Tsimlyansk, Russia	Step	ITCE-1961	Tsvang et al. (1985)
1986	Lövsta, Sweden	Grass	Surface layer	Högström (1990)

These findings were the basis for many famous experiments (Table I), among them many prominent Australian experiments for studying turbulent exchange processes (Garratt and Hicks, 1990), the so-called intercomparison experiments for turbulence sensors (Miyake et al., 1971; Tsvang et al., 1973; Dyer et al., 1982; Tsvang et al., 1985), and finally the KANSAS 1968 experiment (Izumi, 1971; Kaimal and Wyngaard, 1990), which was the basis for the widely used universal functions of Businger et al. (1971). An important state-of-the-art summary of turbulent exchange between the lower atmosphere and the surface was given in 1973 at the Workshop on Micrometeorology (Haugen, 1973).

After some criticism of the experimental design of the KANSAS experiment by Wieringa (1980), and the reply by Wyngaard et al. (1982), several micrometeorological experiments were conducted, including investigations of fundamental micrometeorological issues, like the Swedish experiments at Lövsta (Högström, 1990). Finally, the corrected universal functions by Högström (1988) comprise our most current knowledge.

3. The Obukhov Length

Obukhov (1946) assumed that the parameters g/T_0 (g: gravity acceleration, T_0: surface temperature), v_* (friction velocity), and $q/(c_p\rho)$, with q:

kinematic heat flux, c_p: specific heat, and ρ: air density, describe atmospheric turbulence above the roughness sublayer. All the symbols and equations here are used according to the original papers (also in Section 4; from Section 5 the symbols in current usage appear). In the original papers, and therefore also in this section and also in Section 4, the temperature of dry air is used and not the potential temperature or virtual potential temperature. Only one parameter with the dimension of length is possible to describe these processes – the Obukhov length –

$$L = -\frac{v_*^3}{\kappa \left(\frac{g}{T_0}\right)\left(\frac{q}{c_p\,\rho}\right)} \tag{1}$$

with κ being the von Kármán constant. In the surface layer (Prandtl layer, constant-flux layer), the vertical fluxes were assumed to be constant with height:

$$\overline{w'T'} = \frac{q}{c_p\,\rho} = \text{const}, \tag{2}$$

$$-\rho\overline{v'w'} = \tau = \text{const}, \tag{3}$$

$$v_* = \sqrt{\frac{\tau}{\rho}} \tag{4}$$

with T': fluctuation of temperature, w': fluctuation of vertical wind velocity, v': fluctuation of horizontal wind velocity, and τ: shear stress. Because L was defined by Monin and Obukhov (1954), this length scale was referred to as the Monin–Obukhov length, but after the reprint of Obukhov's (1946) paper in 1971 in Boundary-Layer Meteorology the term Obukhov length (Businger and Yaglom, 1971) became more appropriate, though not always used. Note that also Lettau (1949) found a dimensionless parameter similar to z/L (z: height), but he used instead of the friction velocity different 'mixing velocities' for horizontal and vertical motions.

Obukhov (1946) proposed that the absolute measurement of the shear stress using eddy covariance according to Equation (3) is of fundamental importance for the investigation of the surface layer and for the control of indirect methods such as the profile approach. Therefore, in 1949, Konstantinonov (Obukhov, 1951) developed a wind vane with two hot wire anemometers (90° angle) for the measurement of the friction velocity (Figure 1). This work was contemporaneous with similar investigations by Montgomery (1948) and Swinbank (1951).

The basic interpretation of the Obukhov length is that of a characteristic scale for the thickness of the so-called 'dynamical sublayer' (Obukhov,

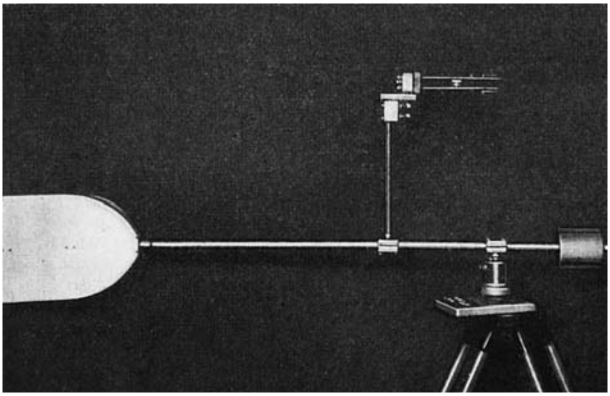

Figure 1. Wind vane with two hot wire anemometers (90° angle) for the measurement of the friction velocity based on Konstantinonov's work in 1949 (Obukhov, 1951).

1946), a layer where the influence of the stratification is negligible (Monin and Yaglom, 1973, 1975). The Obukhov length is proportional to this thickness but not identical. Nowadays, the definition is connected with a dimensional analysis of the equation of turbulent kinetic energy (TKE) and the ratio of the buoyancy and shearing effects (Stull, 1988; Bernhardt, 1995).

4. The Monin–Obukhov Similarity Theory

A.S. Monin (born 2 July 1921) and A.M. Obukhov (5 May 1918 to 3 December 1989), seen in Figure 2, developed their famous similarity theory (Monin and Obukhov, 1954) on the basis of the following findings:

- fundamental experimental work at the Geophysical Main Observatory Leningrad, directed by scientists such as Lajchtman, Budyko, etc.,
- logarithmic wind profile (Prandtl, 1925),
- zero-plane displacement (Paeschke, 1937),
- Obukhov length (Obukhov, 1946).

Using the Buckingham Π-theorem (Kantha and Clayson, 2000) for the dimensionless wind and temperature profiles, Monin and Obukhov argued that the correct dimensionless group for the wind gradient in the surface layer is $\left(\frac{\kappa z}{v_*}\right)\frac{\partial \overline{v}}{\partial z}$ and for the temperature gradient $\left(\frac{z}{T_*}\right)\frac{\partial \overline{T}}{\partial z}$ with $T_* = -\frac{\overline{w'T'}}{v_*}$: dynamical temperature, and both only a function of the parameters g/T_0, v_*, $q/(c_p\rho)$, and the height z. There exists only one dimensionless coefficient z/L. Again, all symbols and equations are used according to the original papers.

Figure 2. (a) A.S. Monin (source: Russian Academy of Science: http://hp.iitp.ru/ger/pers_ac.htm). (b) A.M. Obukhov (source: Yaglom, 1990).

It should be noted that the dimensionless profiles are written without the von Kármán constant κ and with a turbulent Prandtl number $Pr_t = 1$. Monin and Obukhov were aware of Priestley and Swinbank's (1947) assumption for the exchange coefficients of heat and momentum for unstable stratification,

$$\frac{1}{Pr_t} = \frac{K_H}{K_m} > 1, \tag{5}$$

but instead used $K_H = K_m$ because of experimental shortcomings. They also documented a possible modification of their theory. Different exchange coefficients for heat and momentum were later mainly investigated by Australian scientists (Garratt and Hicks, 1990).

In accordance with the dimensionless parameter z/L, the wind and temperature profiles can be written with universal functions:

$$\left(\frac{\kappa z}{v_*}\right) \frac{\partial \overline{v}}{\partial z} = \varphi_1\left({}^{z}\!/_{L}\right), \tag{6}$$

$$\left(\frac{z}{T_*}\right) \frac{\partial \overline{T}}{\partial z} = \varphi_2\left({}^{z}\!/_{L}\right). \tag{7}$$

The universal function was developed as a power series in the case of $|z/L| < 1$ with $\beta = 0.6$:

$$\varphi\left({}^{z}\!/_{L}\right) = 1 + \beta \frac{z}{L}. \tag{8}$$

Currently we use $\beta = 6$ in the case of stable stratification, but in 1954 the experimental data base was too small and basically unable to determine the parameter more exactly.

For strong unstable stratification, $z/L \ll -1$, the assumed power law is:

$$f\left({}^{z}\!/_{L}\right) \approx C\left({}^{z}\!/_{L}\right)^{-1/3} + \text{const.} \tag{9}$$

A new scaling for the strongly unstable case was introduced by Kader and Yaglom (1990).

For strong stable stratification, $z/L \gg 1$, because of

$$K = \kappa\, v_{*}\, L\, Ri \tag{10}$$

Obukhov and Monin found $Ri \approx R = \text{const.}$

Twenty to thirty years ago, the Monin–Obukhov similarity theory was the accepted dogma and it was nearly impossible to publish results in disagreement with the similarity theory (especially in the former Soviet Union). For example, the first studies on countergradient transfer above the ocean (Foken and Kuznecov, 1978) and in forests (Denmead and Bradley, 1985) were published in the grey literature.

Obukhov also derived similarity functions that are absolutely independent of the Monin–Obukhov similarity theory and are not directly related to the work of Monin and Obukhov (1954), e.g., the similarity functions for the temperature structure parameter

$$C_T^2 \approx \left(\overline{w'T'}\right)^{4/3} \left(\frac{g}{T_0}\right)^{-2/3} (z)^{-4/3} \tag{11}$$

can be found in Obukhov (1960).

5. Universal Functions

After the development of the similarity theory for the surface layer, much experimental effort went into determining universal functions; e.g., see the early work of Swinbank (1964, 1968), Tschalikov (1968), Zilitinkevich and Tschalikov (1968), Webb (1970), and Dyer and Hicks (1970); for an overview, see Foken (2003). The widely used universal functions of Businger et al. (1971) are based on observations from the 1968 KANSAS experiment (Izumi, 1971) and are different for momentum, ϕ_{m}, and heat exchange, ϕ_{H}:

$$\varphi_{\mathrm{m}}\left({}^{z}\!/\!_{L}\right)=\left(1-15{}^{z}\!/\!_{L}\right)^{-1/4}, \qquad -2<{}^{z}\!/\!_{L}<0, \tag{12a}$$

$$\varphi_{\mathrm{m}}\left({}^{z}\!/\!_{L}\right)=1+4.7{}^{z}\!/\!_{L}, \qquad 0<{}^{z}\!/\!_{L}<1, \tag{12b}$$

$$\varphi_{\mathrm{H}}\left({}^{z}\!/\!_{L}\right)=0.74\left(1-9{}^{z}\!/\!_{L}\right)^{-1/2}, \quad -2<{}^{z}\!/\!_{L}<0, \tag{13a}$$

$$\varphi_{\mathrm{H}}\left({}^{z}\!/\!_{L}\right)=0.74+4.7{}^{z}\!/\!_{L}, \qquad 0<{}^{z}\!/\!_{L}<1. \tag{13b}$$

Note that the functions are based on $\kappa=0.35$ and $1/Pr_{\mathrm{t}}=1.35$.

During the Workshop on Micrometeorology (Haugen, 1973), the fundamental relationships were also discussed, e.g., the O'KEYPS equation (Obukhov, Kaimal, Elliot, Yamamoto, Panofsky, Sellers – see Panofsky, 1963; Businger, 1988)

$$\left[\varphi_{\mathrm{m}}\left({}^{z}\!/\!_{L}\right)\right]^{4}-\gamma\left({}^{z}\!/\!_{L}\right)\left[\varphi_{\mathrm{m}}\left({}^{z}\!/\!_{L}\right)\right]^{3}=1, \tag{14}$$

which can be solved using the universal function of the Dyer–Businger type (Businger, 1988):

$$\varphi_{\mathrm{m}}\left({}^{z}\!/\!_{L}\right)=\left[1+\gamma\left({}^{z}\!/\!_{L}\right)\right]^{-1/4}=1. \tag{15}$$

The relationship between the universal function of heat and the universal function of momentum is provided by the Dyer–Businger equation:

$$\varphi_{\mathrm{H}}=\varphi_{\mathrm{m}}^{2} \tag{16a}$$

for $z/L<0$,

$$\varphi_{\mathrm{H}}=\varphi_{\mathrm{m}} \tag{16b}$$

for $z/L\geq 0$. An alternative convention of including the turbulent Prandtl number in the definition of the Obukhov length (Equation (1)) is (Yaglom, 1977):

$$L_{\mathrm{Yag}}=-\frac{u_{*}^{3}}{\kappa\left(\frac{1}{Pr_{\mathrm{t}}}\right)\left(\frac{g}{T_{0}}\right)\left(\frac{\overline{w'T'}}{c_{p}\,\rho}\right)}, \tag{17}$$

(u_{*}: friction velocity according to the presently used nomenclature) or the definition without the von Kármán constants, the latter as in the work of S.S. Zilitinkevich. Also, the turbulent Prandtl number can be used either in the profile equation

$$\left(\frac{\kappa}{Pr_{\mathrm{t}}}\right)\left(\frac{z}{T_{*}}\right)\frac{\partial\overline{T}}{\partial z}=\varphi_{\mathrm{H}}\left({}^{z}\!/\!_{L}\right), \tag{18}$$

or in the universal function

$$\varphi_{\mathrm{H}}\left({}^{z}\!/_{L}\right) = Pr_{\mathrm{t}}\left[1+\gamma\left({}^{z}\!/_{L}\right)\right]^{-1/2}, \quad {}^{z}\!/_{L} < 0, \tag{19}$$

as done by Högström (1988).

The Obukhov length was defined for dry air in the original work (Obukhov, 1946; Monin and Obukhov, 1954). The use of the virtual (or sonic) temperature T_{v} is physically more appropriate for moist air. Furthermore, according to Poisson's equation, the temperature must be replaced by the potential temperature Θ to make the Obukhov length independent of the measuring height (Businger, 1988):

$$L = -\frac{u_*^3}{\kappa\left(\frac{g}{\Theta_{v0}}\right)\left(\frac{\overline{w'\Theta_v'}}{c_p\,\rho}\right)}. \tag{20}$$

All universal functions were determined for dry conditions. Up to now, systematic experiments to determine the moisture influence on the universal functions are lacking. Attempts by Panin et al. (1982) have shown that possible influences are within the error bar of the measurements, which is not surprising, because possible influences should be very small if all equations are consistent.

6. Present Status of the Monin–Obukhov Similarity Theory

The criticism of the KANSAS experiment and its unrealistic von Kármán constant by Wieringa (1980) was connected with flow distortion problems of the tower, overspeeding of the cup anemometers, and the unstable performance of the phase-shift sonic anemometers. Responding to Wieringa's findings, Högström (1988) re-formulated the universal functions of Businger et al. (1971). These functions are presently used in the form:

$$\varphi_{\mathrm{m}}\left({}^{z}\!/_{L}\right) = \left(1-19.3\,{}^{z}\!/_{L}\right)^{-1/4}, \qquad -2 < {}^{z}\!/_{L} < 0, \tag{21a}$$

$$\varphi_{\mathrm{m}}\left({}^{z}\!/_{L}\right) = 1+6\,{}^{z}\!/_{L}, \qquad 0 < {}^{z}\!/_{L} < 1, \tag{21b}$$

$$\varphi_{\mathrm{H}}\left({}^{z}\!/_{L}\right) = 0.95\left(1-11.6\,{}^{z}\!/_{L}\right)^{-1/2}, \qquad -2 < {}^{z}\!/_{L} < 0, \tag{22a}$$

$$\varphi_{\mathrm{H}}\left({}^{z}\!/_{L}\right) = 0.95+7.8\,{}^{z}\!/_{L}, \qquad 0 < {}^{z}\!/_{L} < 1. \tag{22b}$$

Note that the functions are based on $\kappa = 0.40$. $Pr_{\mathrm{t}}^{-1} = 1.05$ is used in the universal function.

The status of the Monin–Obukhov similarity theory, including a definition of the conditions for application, was discussed at two conferences/workshops organized by the European Geophysical Society: firstly in 1990 in Copenhagen (Dlugi and Marcart, unpublished) and secondly in 1994 in Grenoble (Foken and Oncley, 1995). Högström (1996) summarized the results of the later workshop on studies of the Monin–Obukhov similarity theory: a value of 0.40 for the von Kármán constant is widely accepted today despite widespread variations in the determined value in the last 50 years (Table II). Even the discussion of a dependency on the roughness Reynolds number (Oncley et al., 1996) seemingly does not withstand close scrutiny because of self-correlating effects (Andreas et al., 2004). According to recent studies, the present accuracy of the turbulent Prandtl number is only 5–10% (see Table III) and data on the turbulent Schmidt number are still missing.

TABLE II

The von Kármán constant according to different authors (Foken, 1990, updated).

Author	κ
Monin and Obukhov (1954)	0.43
Businger et al. (1971)	0.35
Pruitt et al. (1973)	0.42
Högström (1974)	0.35
Yaglom (1977)	0.40
Kondo and Sato (1982)	0.39
Högström (1985, 1996)	0.40 ± 0.01
Andreas et al. (2004)	0.387 ± 0.004

TABLE III

The reciprocal turbulent Prandtl-number according to different authors (Foken, 2003).

Author	Pr_t^{-1}
Businger et al. (1971)	1.35
Correction according to Wieringa (1980)	1.00
Correction according to Högström (1996)	1.05
Kader and Yaglom (1972)	1.15–1.39
Foken (1990)	1.25
Högström (1996)	1.09 ± 0.04

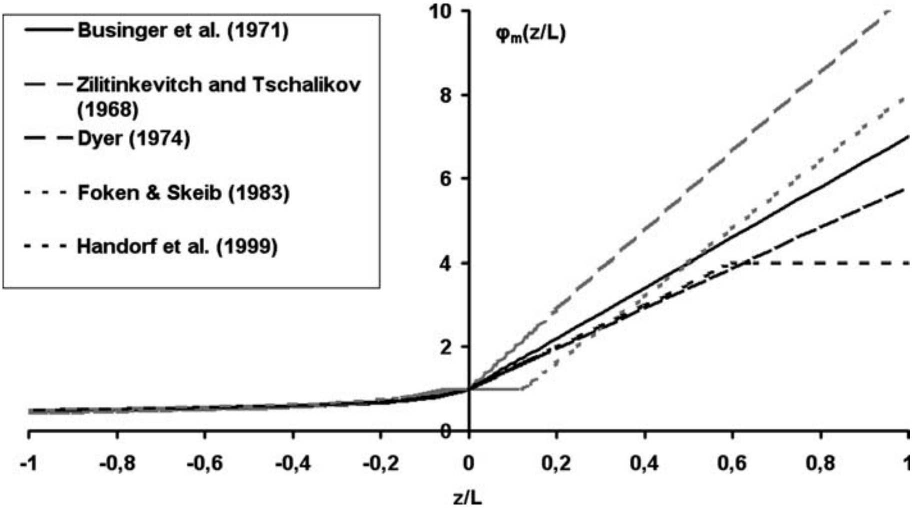

Figure 3. Universal function according to different authors in the modification by Högström (1988).

Högström (1996) gave the following values for the accuracy of the presently used universal functions (Figure 3), which seem to be in agreement under unstable conditions but not under stable conditions:

- $|z/L| \leq 0.5 : |\delta\varphi_{\mathrm{H}}| \leq 10\%$;
- $|z/L| \leq 0.5 : |\delta\varphi_{\mathrm{m}}| \leq 20\%$;
- $z/L > 0.5 : \varphi_{\mathrm{H}},\ \varphi_{\mathrm{m}} = \text{constant}$? E.g., Handorf et al. (1999) found $\varphi_{\mathrm{m}} = 4$ for $z/L > 0.6$;
- $\varphi_{\mathrm{H}}, \varphi_{\mathrm{m}} = \mathrm{f}\ (\mathrm{z_i})$? According to recent results of Johannson et al. (2001), the universal functions depend on the mixed-layer height z_i.

One of the most serious problems for the use of the Monin–Obukhov similarity theory is the roughness sublayer, found above the canopy and up to 2–3 times the canopy height. For vegetated surfaces (canopy height > 0.1 m) these effects cannot be neglected, and were first described in detail by Raupach et al. (1980) in the wind tunnel and by Garratt (1980) over rough natural surfaces based on the Koorin experiment. Under these conditions, the profile equations must be modified using a universal function depending on the height of the roughness sublayer (Garratt, 1992). For tall vegetation or in an urban area, the roughness sublayer may be tens of metres thick and the constant-flux layer, for which the similarity theory is valid, is often very shallow. Furthermore, above high vegetation a mixing layer occurs (Raupach et al., 1996) and the exchange process is partly organized as coherent structures (Högström and Bergstrom, 1996).

Because of the so-called residual in the energy balance closure (Culf et al., 2004) an additional influence on the accuracy of the universal functions occurs when the ‘unclosed’ energy balance is caused by an underestimation of the turbulent fluxes determined with the eddy covariance, while the gradients in Equations (6) and (7) are exactly measured. According to our present knowledge about the energy balance closure problem, the accuracy of the eddy covariance (sensors, corrections, calibrations; for details see Lee et al., 2004) is not a significant factor.

7. Conclusions

‘There are some among us who consider turbulence and its measurement to be a black art. There are others who criticize because they perceive a lack of proof of the validity of the measurements that are reported; and there are some of us who must recognize that some of our earlier results are indeed suspect. However, all is not as bad as it might sometimes seem’ (Hicks, 1986). Progress has been made in the last 15–20 years, e.g., the eddy-covariance method has been extensively updated in the last 5–10 years (new sensors, updated corrections, data quality checks, e.g., Lee et al., 2004).

A better understanding of the limitations of the Monin–Obukhov similarity theory under non-ideal conditions depends upon an exact knowledge of all parameters of the similarity theory. Therefore, we agree with Wyngaard et al. (1982) that the problems with KANSAS 1968 can only be solved with a new experiment.

At the present time, the use of the Monin–Obukhov similarity theory is limited to the surface layer (constant-flux layer) above the roughness sublayer (probably not valid above tall vegetation), to a range of $|z/L| \leq 1$–2, and over homogeneous surfaces. Furthermore, even under ideal conditions, the theory has an accuracy of only about 10–20%.

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