

50 YEARS OF THE MONIN-ObUKHOV SIMILARITY THEORY

Thomas Foken
University of Bayreuth

Reynolds 1894
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Prandtl 1920
Richardson 1920
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Haugen 1973



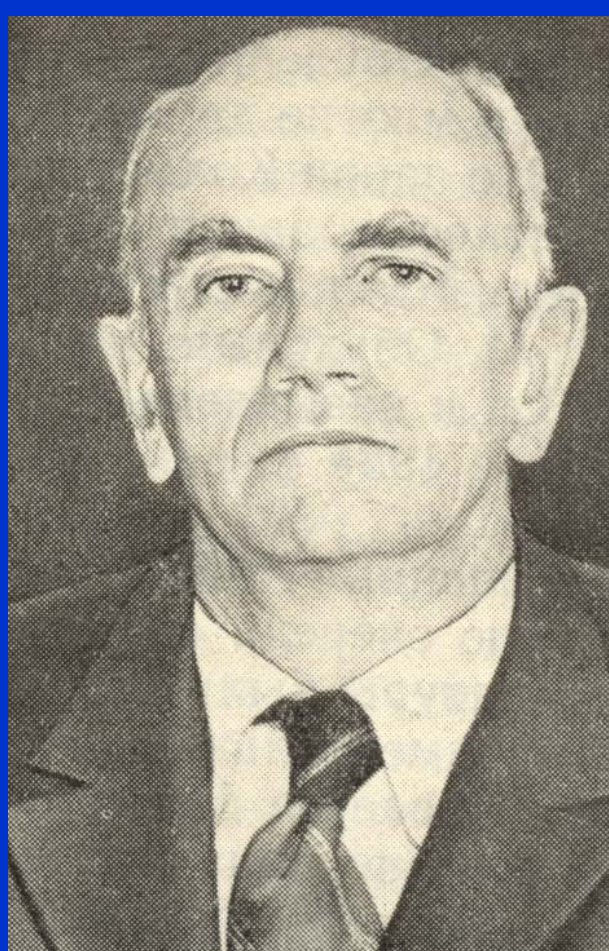
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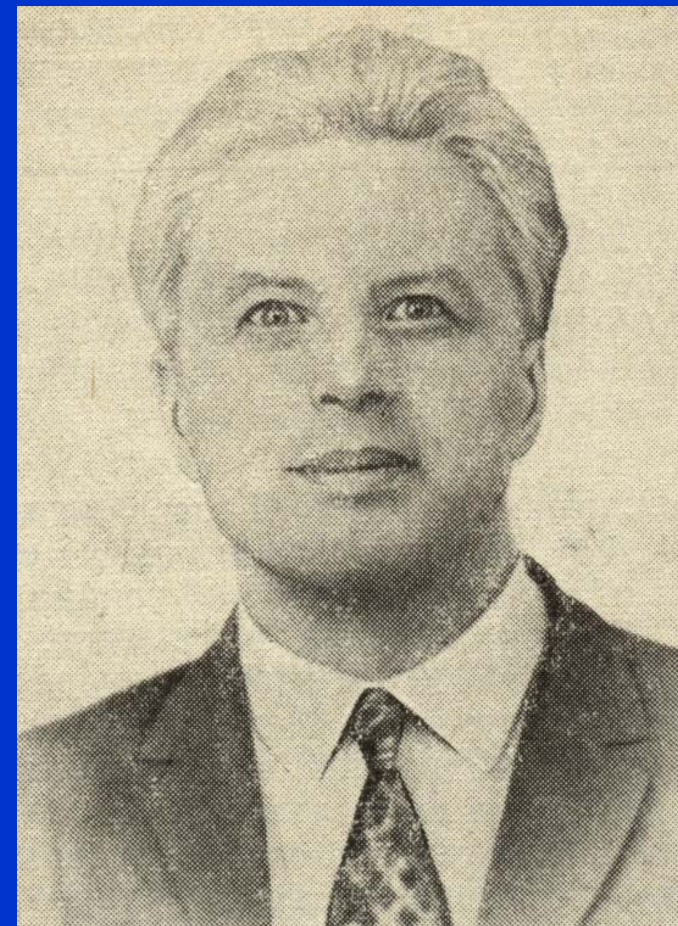


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Aleksander M. Obukhov
05.05.1918 – 03.12.1989

Obituary by A.M. Yaglom:
BLM 53 (1990), v-xi



Andrej S. Monin
*** 02.07.1921**



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Professional Contact with A. M. Obukhov and A. S. Monin

- 1975 and 1976: Member of the KASPEX-75 and KASPEX-76 expeditions of the Institute of Oceanology Moscow (Director: A. S. Monin)
- 1981: Member of the ITCE-81 expedition in Tsimlyansk, Russia of the Institute of Physics of the Atmosphere (Director: A. M. Obukhov)
- 1980 – 1990: Scientific secretary of the KAPG-Project “Atmospheric Boundary Layers” initiated by A. M. Obukhov



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Short History of Micrometeorology

Basic findings about turbulence in hydrodynamics as well as in the atmosphere by Reynolds, Taylor, Prandtl, Richardson, v. Kármán, and others.

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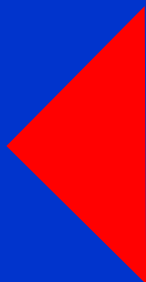
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Before the 2nd World War the centre of micrometeorology was in Vienna, Munich, and Potsdam: Schmidt ('Austausch' coefficient), Geiger (Climate near the ground), Paeschke (Zero-plane displacement), Albrecht (Experimental designs and global energy balance; after the war in Australia), Lettau (Leipzig wind profile; after the war in USA)



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During the 2nd World War most of the findings came from Russian scientists: Kolmogorov (Isotrope turbulence), Obukhov (Similarity analysis, etc.), Yaglom, Monin, and others.



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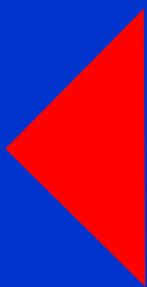
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The Obukhov Length

$$L = - \frac{v_*^3}{\kappa \cdot \frac{g}{T_0} \cdot \frac{q}{c_p \cdot \rho}}$$



$$\overline{w'T'} = \frac{q}{c_p \cdot \rho} = const$$

$$-\overline{\rho u'w'} = \tau = const \quad v_* = \sqrt{\frac{\tau}{\rho}}$$

Obukhov, A. M., 1946: Turbulence of the atmosphere with inhomogeneities in temperature (in Russian). Izv. AN SSSR, vol. 1

All symbols according to the original papers!



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Basis of the Obukhov Length

Assumption that the following
parameters describe the atmospheric
turbulence above the canopy:

$$\frac{g}{T_0} \quad v_* \quad \frac{q}{c_p \cdot \rho}$$

A single parameter with the dimension of
length is possible – the Obukhov Length.

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The Turbulent Prandtl Number

Assumption $K_H > K_m$ for unstable stratification:

$$\frac{1}{Pr_t} = \frac{K_H}{K_m} > 1$$

Remark: Monin & Obukhov (1954) used $K_H = K_m$ because of experimental deficits, but they also documented a modification of their theory.



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The Eddy Covariance Method

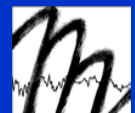
$$\tau = -\rho \overline{u'w'}$$

Obukhov: The absolute measurement of the friction velocity is of fundamental importance for the investigation of the surface layer and for the control of indirect methods.

Montgomery, R. B., 1948: Vertical eddy flux of heat in the atmosphere. *J. Meteorol* 5: 265-274

Obukhov, A. M., 1951: Investigation of the micro-structure of the wind in the near-surface layer of the atmosphere (in Russian). *Izv. AN SSSR, ser. geophys.*, vol. 3, p. 49ff

Swinbank, W.C., 1951: The measurement of vertical transfer of heat and water vapor by eddies in the lower atmosphere. *J. Meteorol.* 8: 135-145.



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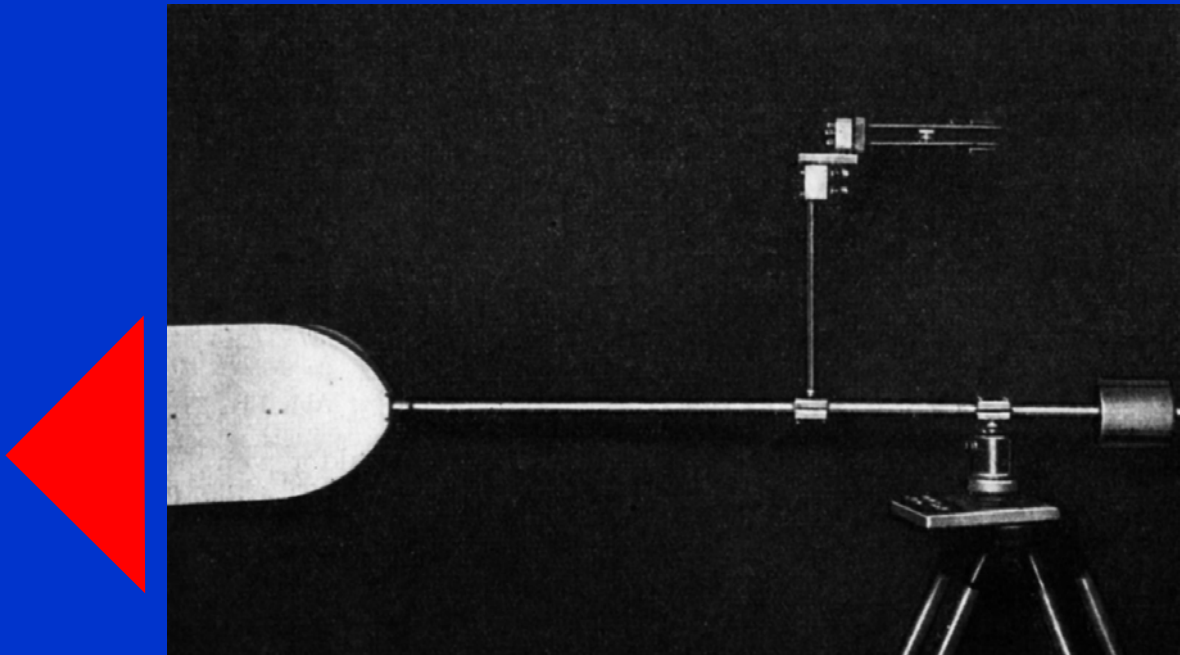
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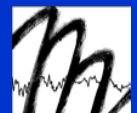


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The Eddy Covariance Method



Wind vane with two hot wire anemometers (90° angle) for the measurement of the friction velocity (Obukhov, 1951) on the basis of Konstantinonov's (1949) work.



The Monin-Obukhov Similarity Theory

Basics:

- Fundamental works of the Geophysical Main Observatory Leningrad: Lajchtman, Budyko, etc.
- Logarithmic wind profile
- Zero-plane displacement (Paeschke, 1937)
- Obukhov length (1946)

Monin, A. S., Obukhov, A. M., 1954: Fundamental laws of the turbulent mixing in the near surface layer of the atmosphere (in Russian). Trudy Geophys. Inst. AN SSSR No. 24 (151), p. 163 ff.

All symbols according to the original papers!

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The Monin-Obukhov Similarity Theory

The dimensionless wind and temperature profile

$$\frac{\kappa \cdot z}{v_*} \cdot \frac{\partial \bar{v}}{\partial z} = \frac{z}{T_*} \cdot \frac{\partial \bar{T}}{\partial z}$$

without κ and $Pr_t =$

must be a function of the external parameters

$$\frac{g}{T_0} \quad v_* \quad \frac{q}{c_p \cdot \rho}$$

and the height z . Only the z/L combination is possible.



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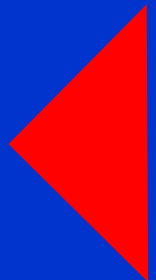
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The Monin-Obukhov Similarity Theory

Accordingly, the wind and temperature profiles with universal functions are:

$$\frac{\kappa \cdot z}{v_*} \cdot \frac{\partial \bar{v}}{\partial z} = \varphi_1\left(\frac{z}{L}\right)$$

$$\frac{z}{T_*} \cdot \frac{\partial \bar{T}}{\partial z} = \varphi_2\left(\frac{z}{L}\right)$$



The universal function can be developed as a power series in the case of $|z/L| < 1$ with $\beta = 0.6$:

$$\varphi\left(\frac{z}{L}\right) = 1 + \beta \frac{z}{L}$$



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The Monin-Obukhov Similarity Theory

Strong unstable stratification $z/L \ll -1$:

$$f(z/L) \approx C(z/L)^{-1/3} + const$$

Strong stable stratification $z/L \gg 1$:
Because of

$$K = \kappa \cdot v_* \cdot L \cdot Ri$$

it follows that:

$$Ri \approx R = const$$



Other Similarity Laws by Obukhov

Similarity functions for the structure parameter:

$$C_T^2 \approx \overline{w'T'}^{4/3} \cdot \left(\frac{g}{T_0} \right)^{-2/3} \cdot z^{-4/3}$$

Obukhov, A. M., 1960: About the structure of the temperature and wind field under convective conditions (in Russian). Izv. A SSSR, ser. geophys., 1392-1396

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The Development of Sonic Anemometers

A tool for the determination of the universal functions:

Bovsheverov, V.M. & Voronov, V.P., 1960. Sonic propeller (in Russian). Izv. AN SSSR, seria geophys 6: 882-885.

Kaimal, J.C. & Businger, J.A., 1963. A continuous wave sonic anemometer-thermometer. J. Climate & Appl. Meteorol., 2: 156-164.

Mitsuta, Y., 1966. Sonic anemometer-thermometer for general use. J. Meteorol. Soc. of Japan, Ser. II, 44: 12-24.



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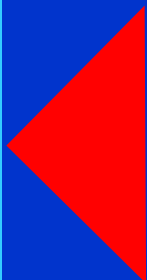
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The Development of Sonic Anemometers



The Russian phase-shift sonic anemometer (more recent type, 1986)



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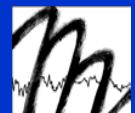
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The Monin-Obukhov Similarity Theory as a Dogma

20-30 years ago it was nearly impossible to publish in reviewed journals papers about results which were not in agreement with the similarity theory (especially in Russia), e.g. the first studies about counter gradients were published in grey literature - above the ocean (Foken & Kuznecov, 1978) - in the forest (Denmead & Bradley, 1985)

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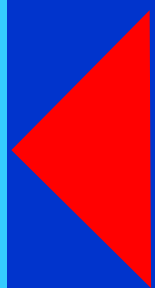
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Experimental Efforts

Different experiments in USA (O'Neill, Lettau), Australia (Kerang, Wangara etc., Swinbank, Dyer), Russia (Tsimlyansk, Tsvang)



Tsimlyansk experimental site



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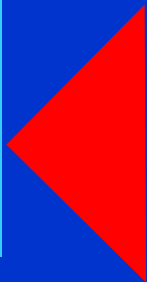


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Experimental Efforts

Comparison experiments for turbulence
 sensors with micrometeorological
 investigations:

Year	Place	Surface	Reference
1968	Vancouver, Canada	water	Miyake et al. (1971)
1970	Tsimlyansk, Russia	step	Tsvang et al. (1973)
1976	Conargo, Australia	step	Dyer (1981); Dyer & Bradley (1982)
1981	Tsimlyansk, Russia	step	Tsvang (1985)



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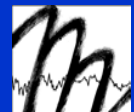
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Experimental efforts

Participants of the ITCE-81 at Tsimlyansk



(from left to right) Perepelkin, Gurjanov, Brömme, Richter, Tsvang, Gerstmann, Zubkovskij †, Obukhov †, Foken, Perepelkina, technician; not on the picture: Kalistratova, Kukharez, Pretel, Zeleny †



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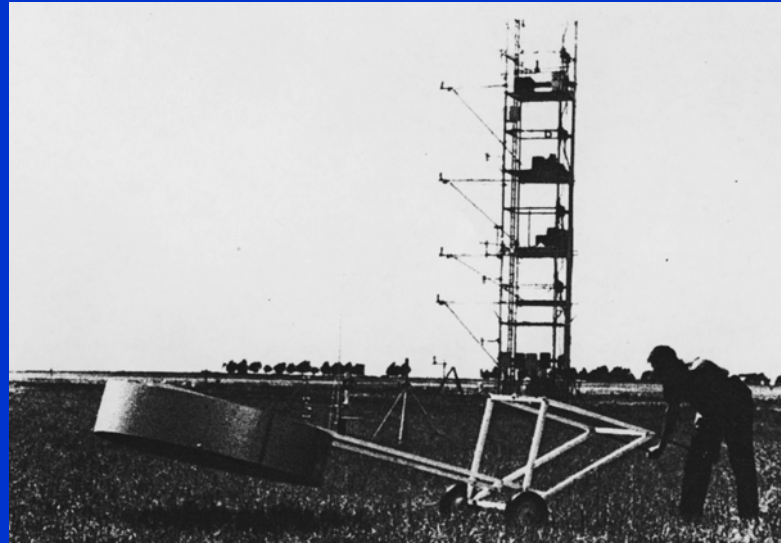
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Experimental Efforts

The KANSAS Experiment 1968



Haugen – Kaimal – Wyngaard – Businger and other:
Izumi, Y., 1971. Kansas 1968 field program data
report. Air Force Cambridge Research Papers, No.
379, Air Force Cambridge Research Laboratory,
Bedford, MA.



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The Universal Function by Businger et al. (1971)

$$\varphi_m\left(\frac{z}{L}\right) = \begin{cases} \left(1 - 15 \frac{z}{L}\right)^{-1/4} & -2 < \frac{z}{L} < 0 \\ 1 + 4.7 \frac{z}{L} & 0 < \frac{z}{L} < 1 \end{cases}$$

$$\varphi_H\left(\frac{z}{L}\right) = \begin{cases} 0.74 \cdot \left(1 - 9 \frac{z}{L}\right)^{-1/2} & -2 < \frac{z}{L} < 0 \\ 0.74 + 4.7 \frac{z}{L} & 0 < \frac{z}{L} < 1 \end{cases}$$

$$\kappa = 0.35 \quad \frac{1}{\text{Pr}_t} = 1.35 \quad !$$

**Businger, J.A., Wyngaard, J.C., Izumi, Y. and
Bradley, E.F., 1971. Flux-profile relationships in the
atmospheric surface layer. J. Atm. Sci. 28: 181-189.**

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The Workshop on Micrometeorology

O'KEYPS equation:

$$\left[\varphi_m\left(\frac{z}{L}\right)\right]^4 - \gamma \cdot \frac{z}{L} \left[\varphi_m\left(\frac{z}{L}\right)\right]^3 = 1$$
$$\varphi_m\left(\frac{z}{L}\right) = \left(1 + \gamma \cdot \frac{z}{L}\right)^{-1/4}$$

Dyer-Businger equation:

$$\varphi_H = \begin{cases} \varphi_m^2 & z/L < 0 \\ \varphi_m & z/L \geq 0 \end{cases}$$

Haugen, D.H. (Editor), 1973. Workshop on micrometeorology. Am. Meteorol. Soc., Boston, 392 pp.

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Normalization of the Obukhov Length

$$L = - \frac{u_*^3}{\kappa \cdot \frac{1}{Pr_t} \cdot \frac{g}{T_0} \cdot \frac{\overline{w'T'}}{c_p \cdot \rho}}$$

Also used with $1/Pr_t$

Also used without κ ,
e. g. S. S. Zilitinkevitch

Yaglom, A.M., 1977. Comments on wind and temperature flux-profile relationships. *Boundary-Layer Meteorol.*, 11: 89-102.

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Normalization of the Universal Functions

$$\frac{\kappa}{\text{Pr}_t} \cdot \frac{z}{T_*} \cdot \frac{\partial \bar{T}}{\partial z} = \varphi_H \left(\frac{z}{L} \right)$$

$$\varphi_H \left(\frac{z}{L} \right) = \text{Pr}_t \left(1 + \gamma \cdot \frac{z}{L} \right)^{-1/2} \quad \frac{z}{L} < 0$$

Use of Pr_t in the profile equation or in the universal function, e.g. in the universal function by Högström (1988)

Yaglom, A.M., 1977. Comments on wind and temperature flux-profile relationships. *Boundary-Layer Meteorol.*, 11: 89-102.

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The Obukhov Length for Moist Air

$$L = - \frac{u_*^3}{\kappa \cdot \frac{g}{T_{v0}} \cdot \frac{w'T_v'}{c_p \cdot \rho}}$$

The use of the virtual (or sonic) temperature is physically more adequate

but

all universal functions were determined for dry conditions.

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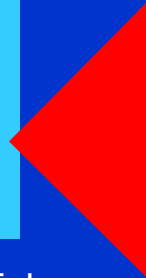
Criticism of the KANSAS Results

- Flow distortion of the tower
- Overspeeding of the cup anemometers
- Problems with the phase shift sonic anemometers
- Unrealistic von-Kármán-constant

Wieringa, J., 1980. A revaluation of the Kansas mast influence on measurements of stress and cup anemometer overspeeding. *Boundary-Layer Meteorol.* 18: 411-430.

Högström, U., 1988. Non-dimensional wind and temperature profiles in the atmospheric surface layer: A re-evaluation. *Boundary-Layer Meteorol.* 42: 55-78.

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The Universal Function by Businger et al. (1971) Modified by Högström (1988)

$$\varphi_m\left(\frac{z}{L}\right) = \begin{cases} \left(1 - 19.3 \frac{z}{L}\right)^{-1/4} & -2 < \frac{z}{L} < 0 \\ 1 + 6 \frac{z}{L} & 0 < \frac{z}{L} < 1 \end{cases}$$

$$\varphi_H\left(\frac{z}{L}\right) = \begin{cases} 0.95 \cdot \left(1 - 11.6 \frac{z}{L}\right)^{-1/2} & -2 < \frac{z}{L} < 0 \\ 0.95 + 7.8 \frac{z}{L} & 0 < \frac{z}{L} < 1 \end{cases}$$

$$\kappa = 0.4 \quad \frac{1}{Pr_t} = 1.05$$

Pr_t in the univ. Fkt. !

Högström, U., 1988. Non-dimensional wind and temperature profiles in the atmospheric surface layer: A re-evaluation. Boundary-Layer Meteorol. 42: 55-78.

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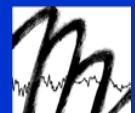


The Recent Status of the Monin-Obukhov Similarity Theory was Discussed e.g.:

- EGS General Assembly 1990 Copenhagen (Mascart & Dlugi)
- EGS Workshop Grenoble 1994 (Foken & Oncley)
- Summary published:

Högström, U., 1996. Review of some basic characteristics of the atmospheric surface layer. *Boundary-Layer Meteorol.*, 78: 215-246.

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The Turbulent Prandtl Number

Author	$1/Pr_t$
Businger et al. (1971)	1.35
– Correction according to Wieringa (1980)	1.00
– Correction according to Högström (1988)	1.05
Kader & Yaglom (1972)	1.15 – 1.39
Foken (1990)	1.25
Högström (1996)	1.09 ± 0.04

from: Foken, Th., 2003: Angewandte Meteorologie, Springer

Remark: Even today the accuracy of the turbulent Prandtl number is only 5-10 %.



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The von-Kármán-Constant

Author	κ
Monin & Obukhov (1954)	0.43
Businger et al. (1971)	0.35
Pruitt et al. (1973)	0.42
Högström (1974)	0.35
Kondo & Sato (1982)	0.39
Högström (1996)	0,40 ± 0,01

from: Foken, Th., 1990: Turbulenter Energieaustausch. Ber. DWD No. 180

Remark: The value of 0.40 for the von-Kármán-constant is widely accepted.



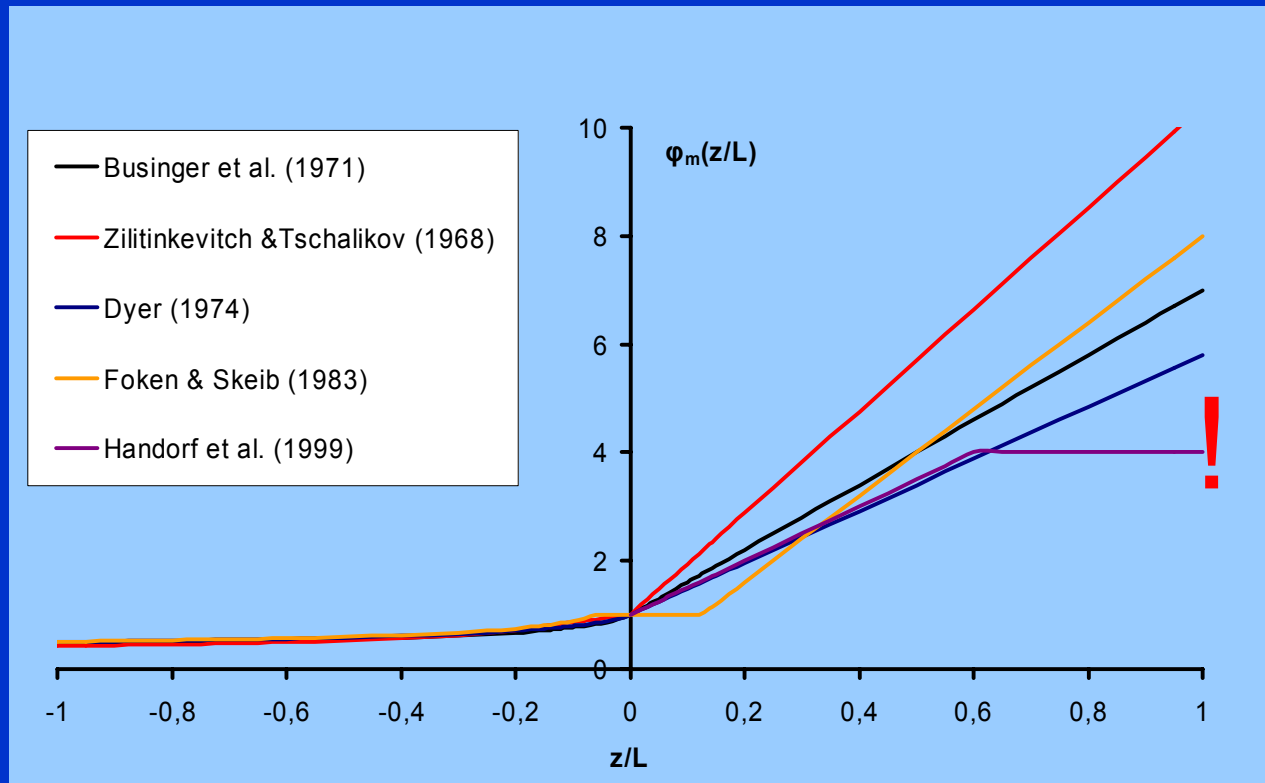
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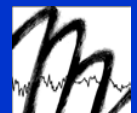


The Universal Function



Remark: After modification by Högström (1988), no significant problems in the unstable case, but ...

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Accuracy of the Universal Functions

- $|z/L| \leq 0.5$: $|\delta\phi_H| \leq 10 \%$
- $|z/L| \leq 0.5$: $|\delta\phi_m| \leq 20 \%$
- $z/L > 0.5$: $\phi_H, \phi_m = \text{const} ?$
- $\phi_H, \phi_m = f(z_i) ?$ (Johannson et al. 2001)

Remark: The limitations in the accuracy of the turbulent Prandtl and Schmidt numbers and of the universal functions are limitations in all weather and climate models!

Högström, U., 1996. Review of some basic characteristics of the atmospheric surface layer. *Boundary-Layer Meteorol.* 78: 215-246.



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Limitation in the Validity of the Monin-Obukhov Similarity Theory

- only valid in the surface layer (constant flux layer)
- only valid for $|z/L| \leq 1 \dots 2$
- only valid above the roughness sublayer (probably not valid above tall vegetation)
- only valid over homogeneous surfaces

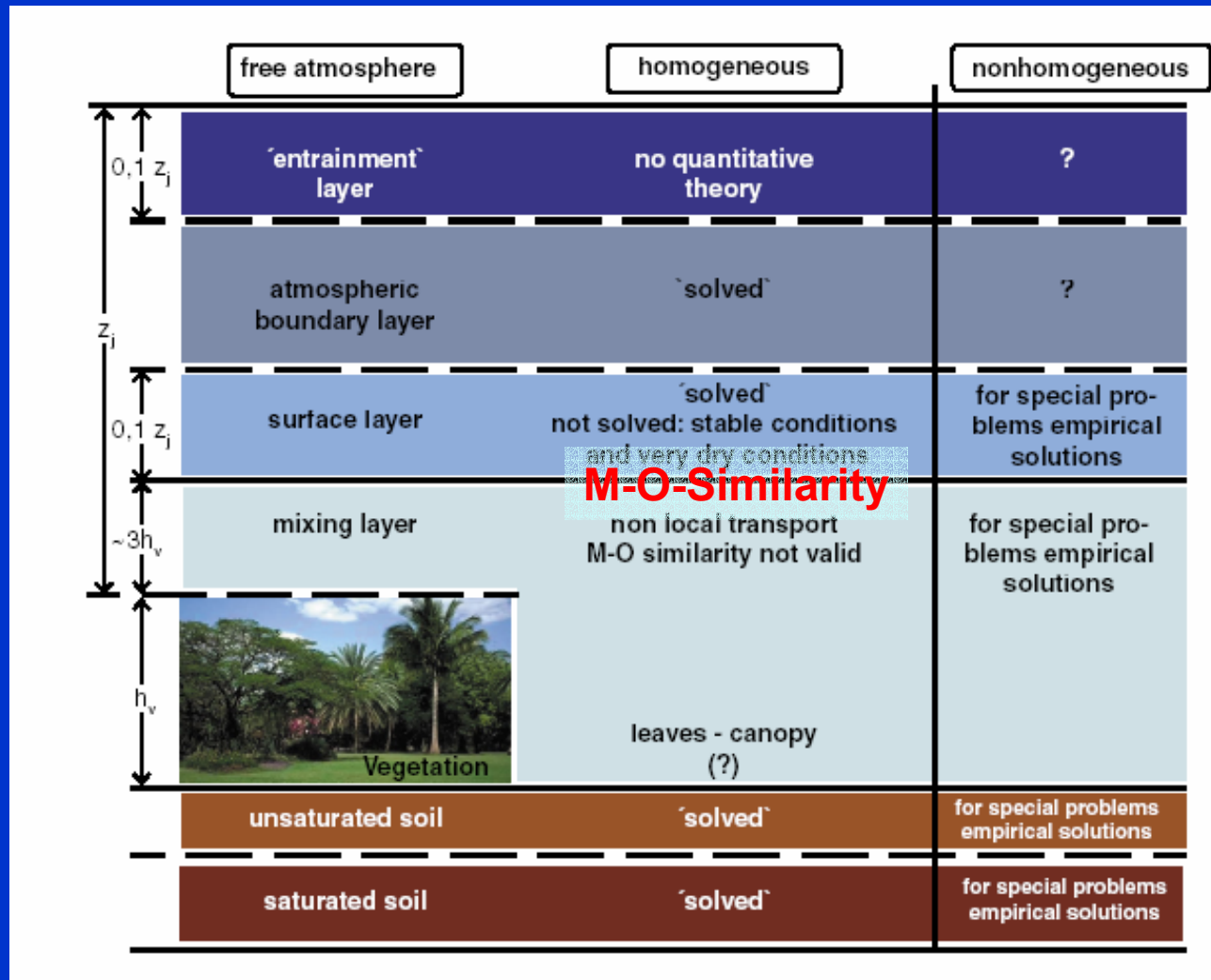
Remark: A better understanding of the limitations and non-ideal conditions depends upon an exact knowledge of all parameters of the similarity theory.



What Do We Know ?

Plugi & Mascart (1990)

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M-O-Similarity



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The key to exact parameters of the similarity theory are direct flux measurements (Obukhov, 1946)

There are some among us who consider turbulence and its measurement to be a black art. There are others who criticize because they perceive a lack of proof of the validity of the measurements that are reported; and there are some of use who must recognize that some of our earlier results are indeed suspect. However, all is not as bad as it might sometimes seem.

B. B. Hicks (1986)

.... and some progress has been made in the last 15-20 years.

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The Time is Ready for a New KANSAS or Tsimlyansk Experiment!

- Wyngaard et al. (1982): The problems with KANSAS 1968 can only be solved with a new experiment.
- The eddy covariance method was highly updated in the last 5-10 years (new sensors, updated corrections, data quality checks).
- Modellers must understand that only a better physics and not 'screws' are the key to better models, and that surface layer fluxes are an important part of each model.



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ITCE-81 Tsimlyansk

**The 50 years of the
Monin-Obukhov
Similarity Theory
also represent 50
years of our
modern
micrometeorology.
Most of our
teachers have
written this story.
We are grateful to
them.**



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